Fakultät für Wirtschaftswissenschaften
Facoltà di Economia
Faculty of Economics and Management

BEMPS –
Bozen Economics & Management
Paper Series

NO 101/ 2023

Monopolistic Duopoly

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MONOPOLISTIC DUOPOLY*

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September 2023

Abstract

We delve into the Hotelling price competition game without assuming full market coverage, and derive three equilibrium configurations. Two of them are well-known: *Hotelling duopoly*, where firms set the prices with the aim of stealing customers from the rival, and the market is fully covered; *Local Monopolies*, where firms avoid strategic interaction and business stealing, and the market is partially covered. In the third, firms interact strategically to keep the market covered, while at the same time avoiding business stealing; we define it as *Monopolistic Duopoly* (MD) because it combines the characterizing features of the other two scenarios. Despite the existence of few contributions on MD, this equilibrium configuration has been substantially ignored. By spelling out the economics of MD and applying its intriguing properties to recent issues, we establish that MD has, instead, relevant implications for the Hotelling literature.

Keywords Hotelling Price Competition Game; Market Coverage; Monopolistic Duopoly.
JEL Codes: L13 (Oligopoly and Other Imperfect Markets); C72 (Noncooperative Games); D21 (Firm Behavior: Theory).

*We wish to thank Luis Corchon, Armando Dominioni, Enrico Minelli, Patrick Rey, Steven C. Salop, and the seminar audience at OLIGO 2023 (Padova), EUREGIO Economics Meeting 2023 (Innsbruck), ASSET 2022 (Rethimno), JEI 2022 (Gran Canaria), EARIE 2022 (Wien), OLIGO 2022 (Nicosia), and ASSET 2021 (Marseille), for useful comments. The usual disclaimer applies.

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1 Introduction

In his seminal paper (Hotelling, 1929), Harold Hotelling provided scholars with a flexible tool to delve into the economics of horizontal product differentiation. While the Hotelling's setup was initially used to analyze the optimal level of differentiation, today the main focus is on competition *given* the differentiation degree. This framework is applied to relevant issues in several fields: not only up-to-date topics in industrial organization, such as platform competition (e.g., Armstrong, 2006; Karle et al., 2020), prominence (e.g., Armstrong and Zhou, 2011; de Cornière and Taylor, 2019), exclusivity (e.g., Carroni et al., 2023; Bedre-Defolie and Biglaiser, 2022), naïvete-based discrimination (Heidhues and Kőszegi, 2017), selection markets (Veiga and Weyl, 2016), and limited consumer attention (Hefti and Liu, 2020), but also, for instance, quality competition in education and health markets (e.g., Brekke et al., 2006; Siciliani and Straume, 2019), labor (Bénabou and Tirole, 2016), and environment (Deltas et al., 2013).

In this paper, we consider the simplest version of the Hotelling framework, featuring two firms located at the extremes of the linear city, zero production costs, linear transportation costs, and price competition. Then, we delve into the price competition game without assuming full market coverage. More precisely, denoting t the marginal transportation cost and v the gross utility from unit consumption, we let t take *any* positive value, for given $v > 0$. In doing so, we derive three different equilibrium configurations. (i) For low values of t relative to v, each firm sets the price with the aim of stealing customers from the competitor. This is the canonical Hotelling duopoly configuration (HD, henceforth), where the market is fully covered and the consumer indifferent between purchasing from either firm obtain positive utility. (ii) For relatively high values of t, each firm sets the price with the aim of avoiding strategic interaction and business stealing. This is the well-known Local Monopolies configuration (LM, henceforth), where the market is partially covered. (iii) For intermediate values of t relative to v, firms sets prices strategically to keep the market covered (like in HD), while at the same time avoiding business stealing (like in LM). We define this lesser-known equilibrium configuration as *Monopolistic Duopoly* (MD, henceforth), where the indifferent consumer obtain zero utility.

To the best of our knowledge, the first analysis containing an equilibrium characterization of MD is due to Salop (1979). Although considering the circular city, instead of the linear one, and a free-entry, zero-profit equilibria framework, the author points out that prices are decreasing in the marginal transportation (and production) costs at the "kinked equilibrium" (corresponding to our MD). Building on Salop (1979), Rey and Salant (2012) explore the optimal behavior of intellectual property owners, which sell licenses to firms evenly located around the circle. The authors show that the upstream supplier sells the exact number of licenses to induce MD as the downstream market equilibrium configuration. Rey and Tirole (2019) employ the Hotelling model –with demand externalities– to demonstrate the range of application of their argument on price caps. In the online appendix, they characterize the three regions we mention above and remark that, under MD, asymmetric price equilibria may arise and prices are strategic substitutes. Strategic substitutability is present also in Thépot (2007).[1] Overall, these papers highlight the following properties of MD that are in stark contrast with those of

[1]A brief analysis of the symmetric MD equilibrium can be found in Cowan and Yin (2008) and Fedele and Depedri (2016). The only other reference to MD we are aware of is an exercise in Chapter 12 of the textbook by Mas-Colell et al. (1995).

HD: (i) a symmetric price equilibrium and a continuum of asymmetric ones arise though firms are symmetric; (ii) prices are strategic substitutes; (iii) prices and profits are decreasing in the marginal transportation cost.[2]

Despite the above findings, the Hotelling literature refers to the full market coverage assumption as a synonym for HD. A natural question is then why MD has been substantially neglected or, at best, relegated to the obscurity of a mathematical curiosum. One possible answer is that its equilibrium properties are believed to be economically uninteresting or, using the wording in Salop (1979), "perverse".

This paper's first contribution is to do justice to MD by carefully exploring the behavior of firms at the MD equilibria and concluding that its properties are far from uninteresting. In particular, we derive two ancillary results that are not reported in the existing papers on MD: (i) at the asymmetric price equilibria, the price difference must be relatively low to avoid profitable unilateral deviations; (ii) at the extremes of the MD parametric interval, only the symmetric equilibrium survives, which implies that the equilibrium prices and profits vary continuously across HD, MD, and LM.

Most importantly, we spell out the intuition as to why the equilibrium configuration changes from LM to MD to HD. To this aim, we refer to the firms' classical trade-off between markups and sales. Because the marginal cost to attract an extra consumer –in terms of price reduction– is increasing in t, firms prefer to set high prices and not the serve the entire market when t is relatively high: LM emerges. As t decreases, the marginal cost reduces, leading to an expansion in sales. This eventually results in all consumers buying and the indifferent consumer obtaining zero utility. At this point, any attempt to gain a further consumer would make that consumer interested in both products: the marginal cost of expanding sales would then jump upwards. To avoid the cost surge, firms adjust the prices to keep the indifferent consumer with zero utility, which explains the equilibrium shift to MD. Put it differently, if one firm lowers the price to attract an extra consumer, the rival firm reacts by increasing the price, thus giving up on that consumer. As t further declines, initially the equilibrium configuration remains MD: because of the discontinuous increase in the marginal cost to expand sales, t has to fall enough to make firms willing to switch to HD.

Firms' MD behavior involves strategic interaction without competition; as such, it evokes the notion of collusion. We argue that the MD behavior is indeed akin to, but distinct from, collusion and refer to it as *quasi-collusion*:[3] on one hand, the (symmetric) MD equilibrium prices maximize the industry profits;[4] on the other hand, they are a Nash equilibrium of a one-shot

[2] Equilibrium configurations with a zero-utility indifferent consumer, therefore similar to MD, have been analyzed in the Hotelling literature on the optimal degree of differentiation too: Economides (1984, 1986) and Hinloopen and Van Marrewijk (1999) build on Lerner and Singer (1937), Smithies (1941), and Vickrey (1964) to explore how such configurations, the "touching equilibria" in their taxonomy, affect the firms' location choice along the segment.

[3] To the best of our knowledge, this term was first used by Papandreou and Wheeler (1954, p. 243) who define quasi-collusion as a situation where "the firms in a market adopt certain standards of behavior which make for parallel or concerted action without overtly arriving at an agreement to do so." This definition is closer to what we currently call *tacit* collusion.

[4] In this respect, Rey and Salant (2012) and Rey and Tirole (2019) point out that the pricing behavior of the firms at the symmetric MD equilibrium replicates that of a two-product monopolist. For an interesting contribution on how the profit-maximizing strategies of a two-product monopolist in the Hotelling framework are affected by the values of t relative to v, see Balestrieri et al. (2021).

game.[5]

Our second contribution is to establish the importance of MD for the Hotelling literature. To this end, we consider three relevant topics, namely platform competition in two-sided markets, asymmetric competition due to prominence or exclusivity, and quality competition in price-regulated markets: we first verify that the existence and the properties of MD are robust to these applications of the basic Hotelling framework; we then derive a set of novel results.

In the two-sided markets application, we show that a MD equilibrium exists if the size of cross-group externalities is large relative to the marginal transportation cost, which causes the nonexistence of the HD equilibrium. The intuition is that relatively high externalities trigger a fierce price war under HD, which leads the platforms to incur losses. By contrast, the MD quasi-collusive behavior allows platforms to avoid such intense competition and reap positive profits. In the asymmetric competition application, we consider policy measures aimed at mitigating the competitive edge given by prominence or exclusivity and prove that the policy effects on consumer surplus can be reversed. For instance, a neutrality policy aimed at limiting biased intermediation can trigger a change in the strategic equilibrium behavior of firms, from HD to MD. In this case, the policy effect turns from negative to positive. Finally, in the quality competition application, we focus on the relationship between the degree of market power and the gap in the equilibrium quality provided by the firms. We find that, under MD, the level of the regulated price affects the sign of such relationship.

The remainder of the paper is organized as follows. Section 2 sets out the model and Section 3 delves into the price competition game. Section 4 discusses MD. Section 5 develops the three applications. Section 6 wraps up the paper and further discusses the robustness of results.

2 The Model

We consider a Hotelling segment of unit length with two firms, indexed by $i \in \{0, 1\}$, located at its extremes: firm 0 is in 0 and firm 1 in 1. A unit mass of consumers is uniformly distributed along the segment. Consumers demand, at most, one unit of a product supplied by the firms. If a consumer located at $x \in [0, 1]$ buys one unit of the product from firm i at price p_i, her utility function is:

$$\mathcal{U}(x, p_i) = v - td_i(x) - p_i, \tag{1}$$

where v is the gross utility from the unit consumption, $d_i(x)$ is the Euclidean distance between the location of the consumer and that of firm i, and t is the marginal transportation cost (or marginal dis-utility of distance). If, alternatively, consumers do not buy, their utility is zero.

We solve equation $\mathcal{U}(x, p_0) = \mathcal{U}(x, p_1)$ for x to obtain the location of the consumer indifferent between purchasing from either firm,

$$x_I(p_0, p_1) = \frac{1}{2} - \frac{p_0 - p_1}{2t}. \tag{2}$$

We also solve equations $\mathcal{U}(x, p_0) = 0$ and $\mathcal{U}(x, p_1) = 0$ for x to derive the location of the consumer

[5] Though not maximizing industry profits, the asymmetric MD equilibria are still reminiscent of collusion because, by the Folk Theorem, collusive equilibria of supergames can be suboptimal.

indifferent between purchasing from firm 0 or firm 1, respectively, and not purchasing,

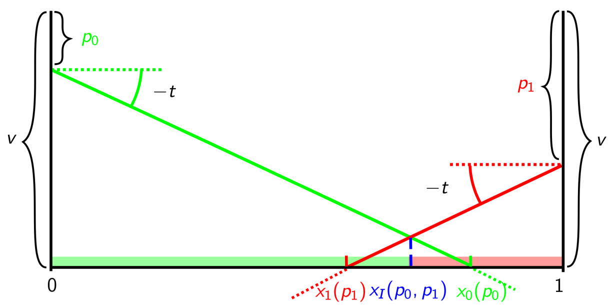
$$x_0(p_0) = \frac{v - p_0}{t} \tag{3}$$

and

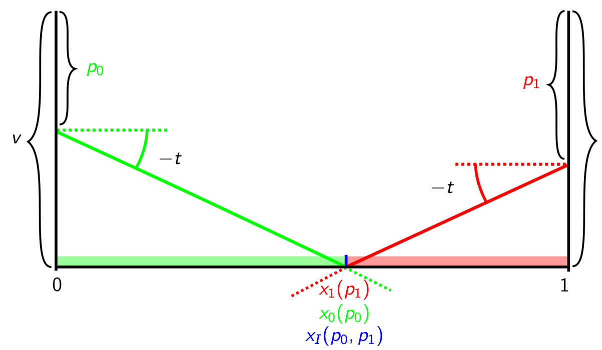
$$x_1(p_1) = 1 - \frac{v - p_1}{t}.$$

Throughout the paper, we refer to x_I as the location of the *indifferent consumer* and to x_i as the location of firm i's *marginal consumer*.

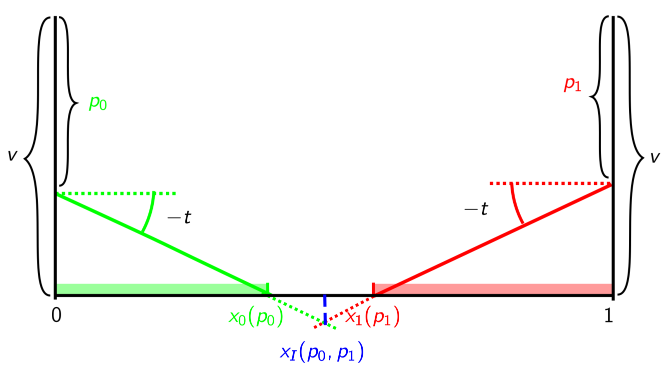
Firms operate a constant-returns-to-scale technology; the marginal cost is normalized to zero. Firms simultaneously choose their prices p_i to maximize the profit function $\pi_i(p_i) = p_i D_i(p_0, p_1)$, where $D_i(p_0, p_1)$ is the demand function. In what follows, we analyze the price competition game without assuming full market coverage; more precisely, we let t take any positive real value, for given $v > 0$.



(a) Hotelling Duopoly.



(b) Monopolistic Duopoly.



(c) Local Monopoly.

Figure 1: Firm 0's price responses.

Focus on firm 0 and consider price $p_1 \in (0, v]$ as given.[6] Then, at most three alternative price responses are available to firm 0, depending on where its marginal consumer, $x_0(p_0)$, lies

[6]With no loss of generality, we disregard intervals $p_1 \leq 0$ and $p_1 > v$, in which firm 1 would reap weakly negative profits, no matter the price chosen by firm 0.

relative to firm 1's one, $x_1(p_1)$.[7]

(i) Firm 0 sets p_0 so that $x_0(p_0) > x_1(p_1)$: see Figure 1a for a graphical representation. This is the standard *Hotelling Duopoly* scenario, where the indifferent consumer x_I locates between x_1 and x_0 and obtains a positive utility. The market is fully covered and firm 0's demand comes from consumers in $[0, x_I]$.

(ii) Firm 0 sets p_0 so that $x_0(p_0) = x_1(p_1)$. Figure 1b illustrates this scenario, which we label *Monopolistic Duopoly*. Here, the locations of x_I and x_i coincide, hence the indifferent consumer obtains zero utility. The market is fully covered, and firm 0's demand comes from consumers in $[0, x_I]$ or, indifferently, in $[0, x_0]$.

(iii) Firm 0 sets p_0 so that $x_0(p_0) < x_1(p_1)$. This is the *Local Monopoly* scenario, depicted in Figure 1c, in which the indifferent consumer locates between x_0 and x_1 and prefers not to buy. The market is partially covered and firm 0's demand comes from consumers in $[0, x_0]$.

Relative to a canonical Hotelling analysis with full market coverage, our approach entails a substantial difference, in that the price levels chosen by firms determine not only the *size* of demands, but also their *functional form*. Figure 2 depicts this point, which Salop (1979) refers to as the "kinked" demand curve: for all $p_0 < 2v - t - p_1$, we have $x_0(p_0) < x_1(p_1)$, so the part of the demand curve represents HD; if the inequality is reversed, we have LM: if, finally, $p_0 = 2v - t - p_1$, we have MD.

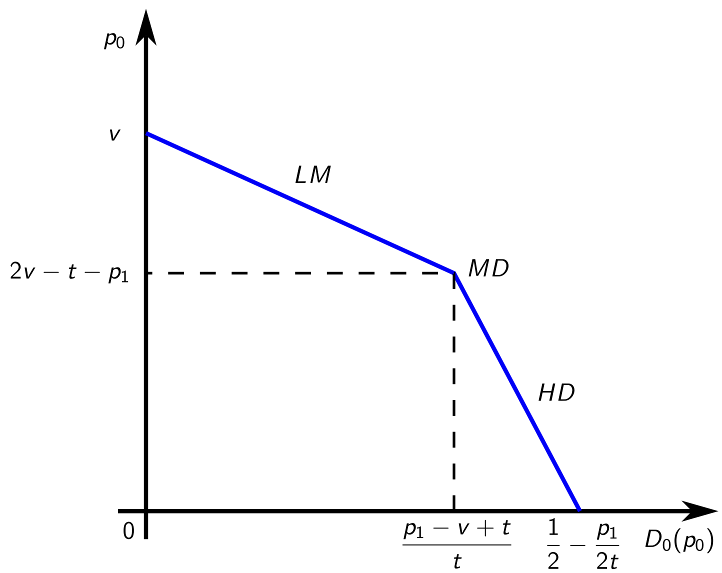


Figure 2: Firm 0's kinked demand function.

3 Analysis

The price competition game is analyzed in two steps. First, in Section 3.1, we derive firm 0's best-response strategy to any $p_1 \in (0, v]$ chosen by the competitor. Next, in Section 3.2, we develop the equilibrium analysis.

[7]For the sake of notational ease, in the following, we will omit the arguments of the functions as long as this does not generates ambiguities.

3.1 Best-response Strategy

To characterize firm 0's best response strategy, we first derive its profit-maximizing price response for each of the three configurations defined in the previous section, and we then compare the resulting profits as a function of p_1. The details are in Appendix A.1.

Optimal price response under HD. For any given $p_1 \in (0, v]$, firm 0 solves the following problem,

$$\max_{p_0} p_0 \times \min\{x_I, 1\} \tag{4}$$
$$\text{s.t. } x_0 > x_1.$$

Note that firm 0's demand function is bounded from above by 1 because, by construction, the indifferent consumer x_I cannot lie outside the unit segment.

Problem (4) describes the following strategic scenario. Let $1 - x_1 = C_1 \in [0, 1)$ be the set of consumers that enjoy nonnegative utility by patronizing firm 1; firm 0, then, sets p_0 low enough to steal some consumers in C_1 from the rival –the set $[x_1, x_I]$, to be precise– as depicted in Figure 1a. The solution to (4) yields firm 0's optimal price response under HD,

$$p_0^{HD}(p_1) = \begin{cases} \frac{p_1+t}{2} & \text{if } 0 < p_1 < \min\left\{3t, \frac{4}{3}v - t\right\} \quad \text{and } t > 0, \\ p_1 - t & \text{if } 3t \leq p_1 \leq v \quad \text{and } t < \frac{1}{3}v. \end{cases} \tag{5}$$

If firm 1 chooses an aggressive pricing strategy, i.e., $p_1 < \min\left\{3t, \frac{4}{3}v - t\right\}$, firm 0 reacts with a low price too, $\frac{p_1+t}{2}$ and its demand is $\frac{p_1+t}{4t} \in (0, 1)$. If firm 1 is more accommodating, i.e., $p_1 \in [3t, v]$, firm 0 can serve the entire market at a higher price, $p_1 - t$, that makes the consumer located in 1 indifferent between purchasing from either firm.

Optimal price response under MD. For any given $p_1 \in (0, v]$, firm 0's problem is

$$\max_{p_0} p_0 \times \min\{x_0, 1\} \tag{6}$$
$$\text{s.t. } x_0 = x_1.$$

Problem (6) describes the following strategic scenario: given p_1 and C_1, firm 0 sets the highest possible p_0 so as to serve all the consumers that are not in C_1, and only them, as illustrated in Figure 1b. The solution to (6) is

$$p_0^{MD}(p_1) = 2v - t - p_1 \text{ if } 0 < p_1 \leq v. \tag{7}$$

Optimal price response under LM. For any given p_1, firm 0 solves

$$\max_{p_0} p_0 \times \min\{x_0, 1\} \tag{8}$$
$$\text{s.t. } x_0 < x_1.$$

Problem (8) describes a scenario where firm 0 sets a relatively high price and avoids strategic interaction with firm 1, as depicted in Figure 1c. A positive measure of consumer exists –the interval (x_0, x_1)– that abstains from consumption. The solution to this problem is given by

$$p_0^{LM}(p_1) = \frac{v}{2} \text{ if } \frac{3}{2}v - t < p_1 \leq v \text{ and } t > \frac{1}{2}v. \tag{9}$$

The LM price response is not defined if $t \leq \frac{1}{2}v$: under this condition, $\frac{3}{2}v - t$ is weakly larger than v and the interval in (9) is empty.

Best-response strategy. For given price p_1 set by the rival, the optimal responses reported in equations (5), (7), and (9) result in different values of firm 0's profit. Comparison of these profits allows us to characterize firm 0's best-response strategy, which we report in the following lemma.

Lemma 1. *Firm 0 best response to any price $p_1 \in (0, v]$ is*

$$p_0^{BR}(p_1) = \begin{cases} p_0^{HD}(p_1) & \text{if } 0 \leq p_1 \leq \min\left\{v, \frac{4}{3}v - t\right\} & \text{and } t > 0, \\ p_0^{MD}(p_1) & \text{if } \frac{4}{3}v - t \leq p_1 < \min\left\{v, \frac{3}{2}v - t\right\} & \text{and } t \geq \frac{1}{3}v \\ p_0^{LM}(p_1) & \text{if } \frac{3}{2}v - t < p_1 \leq v & \text{and } t > \frac{1}{2}v. \end{cases}$$

Proof. See Appendix A.1. □

Lemma 1 shows that firm 0's best response to the competitor's price depends on the relative sizes of parameters t and v that, in turn, govern how firms manage the classical trade-off between markup and sales volumes. In general, for a given v, the smaller t the easier to gain volumes by lowering the price. As t increases, a price cut is gradually less effective in boosting sales. Armed with this intuition, let us start from t small relative to v and gradually increase its value.

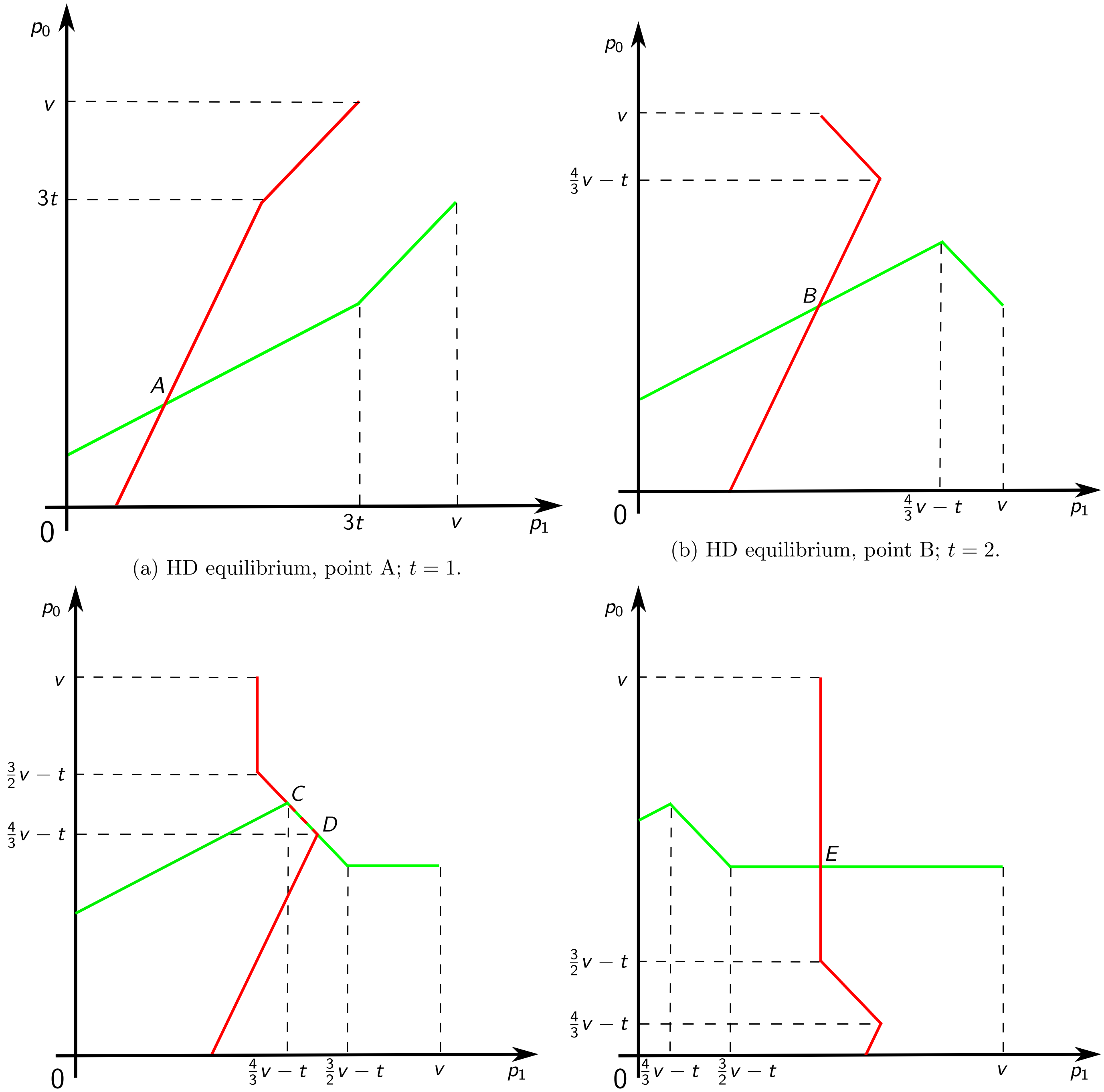
If $t < \frac{1}{3}v$, volumes are very sensitive to price variations. The cost in terms of lost margin to boost sales is small, and firm 0 always behaves as a Hotelling duopolist: its aim is to steal consumers from the rival by setting price(s) $p_0^{HD}(p_1)$. This pricing strategy is graphically represented by the green line in panel 3a of Figure 3.

If $\frac{1}{3}v \leq t \leq \frac{1}{2}v$, volumes turn less sensitive to price variations. As a result, firm 0 still reacts as a Hotelling duopolist if the rival prices aggressively, i.e., $p_1 \in (0, \frac{4}{3}v - t)$. By contrast, firm 0 increases the price to $p_0^{MD}(p_1)$ and shifts to the MD behavior in response to a more accommodating strategy by firm 1, i.e., $p_1 \in [\frac{4}{3}v - t, \min\left\{v, \frac{3}{2}v - t\right\}]$. See the green line in panel 3b.

Finally, if $t > \frac{1}{2}v$, volumes are relatively unresponsive to price variations. Again, firm 0 behaves as a Hotelling duopolist if the rival sets a low p_1 and as a monopolistic duopolist for intermediate p_1. However, if firm 1 sets a high price, $p_1 \in (\frac{3}{2}v - t, v]$, firm 0 prefers to further increase its price to $p_0^{LM}(p_1)$ and get a local monopolist position. See the green line in panels 3c and 3d.

3.2 Equilibrium

The analysis of firm 0's best response function puts us in a position to characterize the set of equilibria of the game, which we report in the following proposition.



(a) HD equilibrium, point A; $t = 1$.

(b) HD equilibrium, point B; $t = 2$.

(c) MD equilibria, segment CD; $t = 3$.

(d) LM equilibrium, point E; $t = 5$.

Figure 3: Best replies of firm 0 (green) and firm 1 (red) and equilibria, $v = 4$.

Proposition 1. *Three alternative equilibrium configurations arise depending on the level of t relative to v:*

(i) if $t < \frac{2}{3}v$, Hotelling Duopoly with symmetric equilibrium prices $p^{HD} \equiv t$;

(ii) if $\frac{2}{3}v \leq t \leq v$, Monopolistic Duopoly with equilibrium prices $p_i^{MD} \equiv v - \frac{t}{2} - k$ and $p_j^{MD} \equiv v - \frac{t}{2} + k$, $k \in [0, k^*]$ and $k^* \equiv \min\{\frac{t}{2} - \frac{v}{3}, \frac{v-t}{2}\}$;

(iii) if $t > v$, Local Monopolies with symmetric equilibrium prices $p^{LM} \equiv \frac{v}{2}$.

Proof. See Appendix A.2. □

Figure 3 offers a graphical representation of the different equilibria. Panels 3a and 3b provide two examples of the HD equilibrium, points A and B. In panel 3c, we represent the MD equilibria, segment CD, and in panel 3d the LM equilibrium, point E.

To grasp the intuition behind Proposition 1, we consider the price reduction firms must incur to attract a further consumer and observe that such reduction crucially depends on whether the market is partially or fully covered.

With partial coverage, a further consumer targeted by firm i is not interested in product j: taking that consumer on board simply requires firm i lowering p_i to guarantee her a nonnegative utility. We label such price reduction as the (marginal) *monopolistic* cost of increasing volumes. Formally, the inverse demand of firm i is $p_i = v - tx_i$ and a marginal increase in sales, dx_i, requires a marginal price drop equal to $\left|\frac{dp_i}{dx_i}\right| = t$.

With full coverage, an extra consumer targeted by firm i is also considering firm j's product: stealing that consumer from the rival requires firm i leaving her at least with the utility she would enjoy by consuming product j and, at the same, triggers firm j's reaction. We refer to the overall price drop as the (marginal) *competitive* cost of increasing volumes. Formally, firm i's demand is $x_i = \frac{1}{2} - \frac{p_i - (\frac{p_i+t}{2})}{2t}$; the inverse demand is then $p_i = 3t - 4tx_i$ and the marginal competitive cost amounts to $\left|\frac{dp_i}{dx_i}\right| = 4t$.

This clarified, we discuss Proposition 1 starting by the LM case, which obtains if $t > v$. Because t is relatively high, attracting a further consumer requires a large price reduction. Accordingly, firms prefer to set relatively high prices, which results in equilibrium partial coverage. Condition $t > v$ is an intuitive one: even if the firms price at marginal cost –zero, in our model– the utility of their farthest potential consumer (i.e., that at the opposite end of the interval) is $v - t < 0$. This individual and, by continuity, some others in her neighborhood, never consider the firm at the opposite end of the interval. Consequently, firms take advantage of their monopoly power on a set of nearby consumers and prefer not to serve the entire market.

When the parametric condition for LM does not hold, i.e., $t \leq v$, marginal-cost pricing turns effective to make each product attractive also for the farthest consumers, which leads to full coverage at equilibrium. If t is relatively high, i.e., $t \in [\frac{2}{3}v, v]$, firms avoid the upwards jump in the cost to expand volumes, from t to $4t$, by adopting a MD behavior. Here, the cost for firm i to attract an extra consumer is still t because firm j, by increasing p_j, does not compete for that consumer. A striking feature of MD is that firms interact strategically, like in HD, but do not compete to steal business from each other, like in LM.

As t decreases, the gap between the competitive and the monopolistic costs shrinks because consumers are increasingly sensitive to the price; at the same time, the MD equilibrium prices rise. If t is sufficiently low, i.e., $t < \frac{2}{3}v$, each firm finds it profitable to accept the competitive cost and sets a lower price than the MD equilibrium price to attract consumers from the rival. This results in the canonical HD competition.

In conclusion, it is worth remarking that the upwards jump in the cost to expand volumes explains why MD exists in a whole interval of values of t, rather than at a single point, as one might expect by inspecting Figures 1 and 2. Because of such discontinuity, t has to decrease enough to make the competitive cost (relatively) acceptable.

To complete our analysis, in Figure 4 we draw the HD, MD and LM equilibrium prices as a function of t. In the MD region, the dashed line represents the symmetric equilibrium; instead, every pair of prices vertically equidistant from the dashed line represents a pair of asymmetric equilibrium prices, provided that they lie within the shaded area.

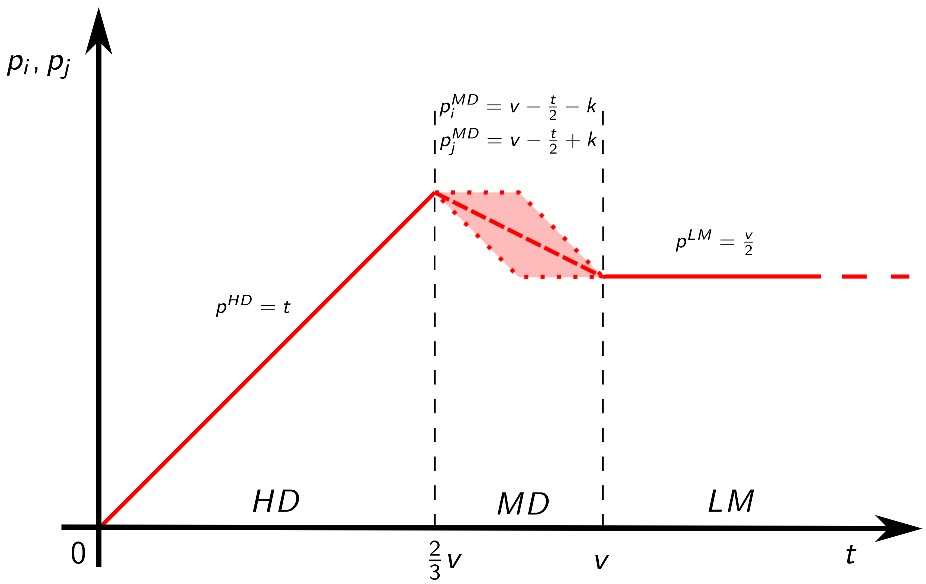


Figure 4: Optimal prices as a function of t.

4 Monopolistic Duopoly: Discussion

MD distinguishes from the canonical HD, and its peculiar characteristics deserve further explanation. We elaborate on such characteristics in what follows.

Remark 1. *Under MD, asymmetric prices can arise at equilibrium, even though firms are symmetric.*

Inspection of Figure 3c reveals that MD best replies $p_i^{MD}(p_j)$ may overlap in the $(p_0,\, p_1)$ plane, whereby a continuum of asymmetric price equilibria may exist. The multiplicity of equilibria stems from the fact that firms aim to avoid competition, while keeping the market just covered. If one firm sets a low (high) price and serves more (less) than half of the market, the other firm adopts a mirror strategy by setting a high (low) price so as to serve the smaller (larger) remaining set of consumers.

Interestingly, the degree of asymmetry is bounded from above by $k^* = \min\{\frac{t}{2} - \frac{v}{3}, \frac{v-t}{2}\}$. To see why, we observe that the binding threshold is $\frac{t}{2} - \frac{v}{3}$ if $t < \frac{5}{6}v$, while it is $\frac{v-t}{2}$ if $t > \frac{5}{6}$; moreover, the switching value $t = \frac{5}{6}v$ lies in the middle of the MD segment $\frac{2}{3}v \leq t \leq v$.

With this in mind, let us start by the sub-interval $t < \frac{5}{6}v$, and suppose k exceeds $\frac{t}{2} - \frac{v}{3}$. In such a case, firm j setting the higher price has a profitable deviation to a lower price, which leads to HD. The intuition is as follows. Because the parameter constellation is closer to HD than to LM, firm j might be tempted to accept the competitive cost of expanding the volumes and, accordingly, cut its price. However, k must be relatively large for such deviation to be profit-increasing; in this case, p_j^{MD} would be so large that volumes would be unprofitably small. Note that the threshold $\frac{t}{2} - \frac{v}{3}$ is zero at $t = \frac{2}{3}v$, in which case the only MD equilibrium is the symmetric one, and rises with t: an increasing t makes firm j's price cut increasingly costly.

A mirror reasoning explains the case $t > \frac{5}{6}v$, where the binding cutoff is $\frac{v-t}{2}$. If k exceeds that value, the lower-price firm i has the incentive to deviate to a higher price, resulting in LM. Here, the parameter constellation is closer to LM than HD, and a deviation to a higher price occurs when $k > \frac{v-t}{2}$ because p_i^{MD} would be unprofitably low. Note that the threshold $\frac{v-t}{2}$ decreases with t and becomes zero at $t = v$, where only the symmetric prices survive at the MD equilibrium.

Remark 2. *Under MD, prices are strategic substitutes.*

This result is in sharp contrast with the canonical case of HD. To give an intuition, recall that the indifferent consumer's utility is positive at the HD equilibrium, but zero at the MD equilibria. Consider then HD and suppose firm i increases its price: firm i's farthest consumers shift to firm j that, in turn, can increase its price and enjoy a larger markup with a nonlower demand. Under MD instead, an increase in p_i does not benefit firm j's demand because the farthest share of firm i's consumers stop buying. Firm j's best response is then to cut p_j to serve these consumers and only them. By referring to the discussion after Proposition 1, firm j can expand volumes at the monopolistic cost, rather than the competitive one.

Consider now a drop in the price set by firm i, which erodes the rival's demand. Under HD, firm j accepts the competitive cost to expand the sales and then cuts p_j to win consumers back. Under MD instead, firm j is unwilling to bear the competitive cost, thus it raises p_j to target the consumers still uninterested in product i, and them only.

Remark 3. *Under MD, equilibrium prices and profits decrease with t.*

Universal agreement exists that t captures the degree of product differentiation and market power in the Hotelling setup. Accordingly, a higher t is associated with milder competition, higher prices, and higher profits. Remark 3 points out that this is not a general property of such a setup. The opposite happens under MD because each firm desires to serve all those consumers who do not patronize the rival, and them only. If t increases, the prices must suitably decline to continue attracting the most distant consumers, and profits shrink too. As a consequence, the marginal transportation cost should be interpreted as an inverse, rather than direct, measure of market power under MD.

Remark 4. *Under MD, firms adopt a quasi-collusive behavior.*

As clarified in the previous section, MD incorporates characteristics of both strategic behavior – prices are interdependent to keep the market covered – and of monopolistic behavior – firms do not aim to steal customers from each other. This points to the role of MD in bridging the competitive HD environment and the nonstrategic LM scenario. As mentioned in the introduction, the idea of strategic interaction without competition evokes the notion of collusion. In what follows, we explain why firms' MD behavior is similar, but not equivalent, to collusion.

Consider the full market coverage region $t \leq v$ and observe that the industry profit-maximizing price is $v - \frac{t}{2}$; clearly, this is also the best collusive price. Yet, $v - \frac{t}{2}$ is not a Nash equilibrium under HD, hence sustaining it would require a suitable supergame setup. By contrast, under MD, the firms choose the industry profit-maximizing price at the Nash equilibrium of the one-shot game. As for the asymmetric MD equilibrium prices, they do not maximize industry profit but are still reminiscent of the possibility of collusion on Pareto-dominated outcomes, as *per* the Folk Theorem.

Interestingly, the analysis of algorithmic pricing has recently pointed out that higher-than-Nash-equilibrium prices can emerge without a proper reward-punishment setup: see, e.g., Calvano et al. (2023), who refer to this sitation as *spurious* collusion.

Remark 5. *Under MD, the total surplus is maximum in the symmetric equilibrium, but the consumer surplus is higher in an asymmetric equilibrium.*[8]

The result on total surplus is driven by the fact that the symmetric MD equilibrium maximizes industry profits and, at the same time, minimizes the transportation costs. Nevertheless, consumer surplus is higher under asymmetry because more than 50% of consumers pay a lower price than the symmetric one, which outweighs the increase in transportation costs.

5 Applications

Our analysis points out that MD is a competitive scenario that emerges in a nontrivial parametric constellation of the Hotelling model. Further, its characteristics are far from "perverse" and can be seen as a "hybridization" between full-fledged duopolistic competition and local monopolies. In this section, we claim that (the intriguing features of) MD may have a relevant impact on the analyses built around the Hotelling setup. To substantiate our point, we modify the model in Section 2 and analyze three up-to-date topics: price competition in two-sided markets, asymmetric competition due to prominence or exclusivity, and quality competition in price-regulated markets.

5.1 Price Competition in Two-Sided Markets

In a growing number of markets, competing platforms manage interactions among different groups of agents. Since the seminal paper by Armstrong (2006), the Hotelling game has become the workhorse for analyzing competition between horizontally differentiated platforms in multi-sided markets. Crucially, Armstrong (2006) finds that the degree of platform differentiation must be high relative to the strength of cross-group externalities to ensure that the platforms' equilibrium profits are positive: see condition (8) on p. 674 of his paper.

However, an increasing number of real-world instances exist where ignoring low-differentiation scenarios might be problematic. A case in point is digital markets with multi-sided platforms. Transportation costs in their traditional, spatial sense are negligible in these markets. In addition, the –possible– interpretation of relatively high transportation costs as a predominant idiosyncratic attachment to a platform may be unsatisfying; indeed, finding the desired content or products on, say, streaming or marketplace platforms is likely no less important than the abstract preference for one platform.

In what follows, we show that a MD equilibrium exists even if transportation costs are small relative to externalities, namely in the region where the HD equilibrium fails to exist. To make our point, we develop a simplified version of Armstrong (2006), in which platforms set prices

[8] Similar normative considerations can be found in Rey and Tirole (2019), Footnote 39, p. 3041.

only on the consumer side.[9] Two platforms indexed $i \in \{0, 1\}$ are located at the endpoints of a Hotelling segment of unit length and mediate the interaction between consumers and producers; for simplicity, the platforms' production costs are normalized to zero.

A continuum of consumers is uniformly distributed along the line, each identified by her location x. Consumers can decide not to be active in the market (zero-home) or to join one platform (single-home). In the former case, the utility is zero. In the latter, the utility function of a consumer located at x and joining platform i is:

$$\mathcal{U}(x, p_i, \mathbb{E}(n_i)) = v + \alpha \cdot \mathbb{E}(n_i) - td_i(x) - p_i. \tag{10}$$

Compared to the utility function (1), the additional term $\alpha\mathbb{E}(n_i)$ captures the cross-group externalities enjoyed by the consumers on platform i: $\alpha > 0$ is the value of the interaction between a consumer and a producer; $\mathbb{E}(n_i)$ is the mass of producers the consumer expects to be active on platform i.

On the other side of the market, each of a continuum of producers is free to choose not to participate in the market (zero-home), to join one platform (single-home), or to join both platforms (multi-home). Producers bear heterogeneous setup costs f to operate in either platform; such costs are uniformly distributed over $[0, 1]$. Producers enjoy the cross-group externality $\gamma > 0$ when interacting with a consumer, and pay a fixed fee, which we normalize to zero, to join each platform.

A cost-f producer is willing to join platform i if and only if $\gamma \cdot \mathbb{E}(D_i) \geq f$, where $\mathbb{E}(D_i)$ is the mass of consumers the producer expects to be active on platform i. The total mass of producers active on platform i is therefore given by $n_i = Prob(f \leq \gamma \cdot \mathbb{E}(D_i))$. Under the assumption of uniform distribution of setup costs, we get $n_i = \gamma \cdot \mathbb{E}(D_i)$.

The timing of the game is as follows. First, the platforms simultaneously set prices p_i to maximize profits, $\pi_i = p_i D_i$. Second, consumers and producers simultaneously make their joining decisions. Here, we are interested in characterizing the equilibrium partition, particularly the parameter constellation where MD is the only equilibrium. Accordingly, we restrict our attention to the symmetric equilibrium in the MD case.

Result 1. *Consider a two-sided market and let $e \equiv \alpha\gamma$ denote the strength of cross-group externalities. Four alternative equilibrium configurations arise depending on the level of t relative to v:*

(i) if $t \leq e$, Monopolistic Duopoly with equilibrium prices $p^{MD} \equiv v - \frac{t}{2} + \frac{e}{2}$;

(ii) if $e < t < e + \frac{2}{3}v$, Hotelling Duopoly with equilibrium prices $p^{HD} \equiv t - e$;

(iii) if if $e + \frac{2}{3}v \leq t \leq e + v$, Monopolistic Duopoly with equilibrium prices $p^{MD} \equiv v - \frac{t}{2} + \frac{e}{2}$;

(iv) if $t > e + v$, Local Monopolies with equilibrium prices $p^{LM} \equiv \frac{v}{2}$.

Proof. See Appendix B. □

[9] More precisely, we rely on Armstrong (2006) to model the consumer side and on Hagiu (2006) to model the producer side. For a similar framework, see Rasch and Wenzel (2013).

The message conveyed by the foregoing result is twofold. On the one hand, it ensures that the findings of Proposition 1 extend to the case of two-sided markets; it is indeed straightforward that points (ii)-(iv) parallel those of Proposition 1. On the other hand, a significant difference emerges. While in one-sided markets the HD equilibrium exists for t arbitrarily close to 0, in the two-sided environment the externalities put downward pressure on the HD equilibrium prices $p^{HD} \equiv t - e$, hence the degree of differentiation must be relatively large, $t > e$, to guarantee equilibrium existence.[10]

However, point (i) in Result 1 is novel: even if t is lower than e, equilibrium interaction among firms exists in the form of MD. What is the intuition behind this outcome? In two-sided markets with positive cross-group externalities, a relatively low t would trigger a fierce price war if platforms' goal were to steal consumers from one another, as is the case under HD: this would lead the platforms to incur losses. By contrast, under MD, platforms want to serve all the consumers that do not patronize the rival and them only. As argued above, this results in quasi-collusive behavior that allows platforms to circumvent competition and earn positive profits.

5.2 Asymmetric Competition: Prominence and Exclusivity

It is often observed that third parties give some firms an edge over competitors. Two common examples are (i) *prominence* and (ii) *exclusivity*. (i) Consumers incurring nontrivial search costs when looking for a deal tend to consider first the sellers which, for instance, are recommended by an intermediary, or are prominently displayed by a search engine. Prominence is then the situation in which those sellers enjoy an advantage over competitors because they are likely to be encountered first by consumers (e.g., Armstrong and Zhou, 2011, Bar-Isaac and Shelegia, 2022, and Ciotti and Madio, 2023). Other examples may include a shopping mall putting a store in a prime position, or a two-sided platform that is also active in one side of the market and steers consumers toward its own products at the detriment of hosted sellers. (ii) Exclusivity is, instead, a contractual clause that gives a specific firm the exclusive right to use some products or production factors. When products/factors are of superior quality, the firm can offer a better product to its consumers (e.g., Carroni et al., 2023). One can think of a sports league selling exclusive broadcasting rights to a pay-TV firm.

The use of the Hotelling game to analyze prominence and exclusivity is increasingly widespread; as for prominence, see, e.g., Armstrong and Zhou (2011) and de Cornière and Taylor (2019); as for exclusivity, see, e.g., Bedre-Defolie and Biglaiser (2022) and Martimort and Pouyet (2022). Among other issues, these papers investigate the conditions under which the competitive edge conferred by prominence or exclusivity is socially desirable and, if not, how to mitigate it.

In what follows, we parsimoniously incorporate asymmetric competition, due to prominence or exclusivity, into the baseline model developed in Section 2 and derive two results. First, we verify that Proposition 1 is robust to this extension. Second, we show that the welfare effects of two policy interventions studied in the literature can be *reversed* if the analysis includes MD.

Suppose a fraction $\mu \in (0, 1]$ of consumers obtains an extra utility $a > 0$ only when consuming the variety supplied by firm 0 and label them type-a consumers. Accordingly, the utility function

[10] Condition $t > e$ is akin to inequality (8) in Armstrong (2006).

of a type-a consumer located in $x \in [0,1]$ is as in (1) if she buys from firm 1, and

$$\mathcal{U}(x, p_0, a) = v + a - tx - p_0$$

if she purchases from firm 0. The remaining fraction $1 - \mu \in [0, 1)$ of consumers are labeled standard because their utility is as in (1) if purchasing from either firm.

The extra-utility a captures the advantage enjoyed by firm 0 vis-à-vis firm 1 thanks to prominence or exclusivity (and perceived as such only by type-a consumers). We will provide more precise interpretations of this extra utility when analyzing the welfare effects of bans on prominence and exclusivity.

We solve the price competition game, restricting our attention to the symmetric equilibrium in the MD case. We also assume that firms do not price discriminate between the two types of consumers. As a consequence, for given prices p_0 and p_1, the indifferent standard consumer enjoys lower utility than the indifferent type-a consumer. With abuse of notation, we denote the former consumer by x_I and the latter by x_I^a.

With that in mind, we derive the following result.

Result 2. *Consider asymmetric competition. Five alternative equilibrium configurations arise depending on the level of t relative to v:*

(i) if $t < \frac{2}{3}v$, firms set the equilibrium prices $p_0^{HD} \equiv t + \frac{1}{3}a\mu$ and $p_1^{HD} \equiv t - \frac{1}{3}a\mu$ and the indifferent consumers x_I and x_I^a obtain positive utility (HD);

(ii) if $\frac{2}{3}v \leq t \leq 2\frac{2-\mu}{4-\mu}v$, both firms set the equilibrium price $p^{MD} \equiv v - \frac{t}{2}$, x_I obtains zero utility (MD), while x_I^a obtains positive utility (HD);

(iii) if $2\frac{2-\mu}{4-\mu}v < t < 2\frac{2-\mu}{4-\mu}v + \frac{4-3\mu}{4-\mu}a$, firms set the equilibrium prices $p_0^{LM} \equiv 2\frac{4-\mu(5-\mu)}{(4-3\mu)(4-\mu)}v + \frac{\mu[(4-\mu)t+(4-3\mu)a]}{(4-3\mu)(4-\mu)}$ and $p_1^{LM} \equiv 2\frac{4-\mu(5-\mu)}{(4-3\mu)(4-\mu)}v + \frac{\mu[(4-\mu)t-(4-3\mu)a]}{(4-3\mu)(4-\mu)}$, x_I obtains negative utility (LM), while x_I^a obtains positive utility (HD);

(iv) if $2\frac{2-\mu}{4-\mu}v + \frac{4-3\mu}{4-\mu}a \leq t \leq v + (1 - \frac{\mu}{2})a$, both firms set the equilibrium price $p^{MD'} \equiv v - \frac{t-a}{2}$, x_I obtains negative utility (LM), while x_I^a obtains zero utility (MD);

(v) if $t > v + (1 - \frac{\mu}{2})a$, firms set the equilibrium prices $p_0^{LM'} \equiv \frac{v+a\mu}{2}$ and $p_1^{LM'} \equiv \frac{v}{2}$ and the indifferent consumers x_I and x_I^a obtain negative utility (LM).

Proof. See Appendix C.1. □

Result 2 shows that, qualitatively, the findings in Proposition 1 extend to the case of asymmetric competition with prominence/exclusivity. In particular, MD emerges for intermediate values of t relative to v. The only remarkable difference is that MD concerns one of the consumer types at a time. This is intuitive because MD leaves the indifferent consumer, either standard or type-a, with zero utility and, as remarked, the indifferent type-a consumer always enjoys larger utility than the standard one. On top of that, Result 2 allows us to argue that its specific features have relevant implications for policy assessments. To see how, we consider the effects of two policy interventions on consumer surplus: neutrality and ban on exclusivity.

Neutrality policy. In line with de Cornière and Taylor (2019), suppose the extra-utility a is due to an intermediary that is integrated with firm 0 and biases its advice in favor of that firm. In this case, $\mu < 1$ represents the share of uninformed consumers who rely on the biased intermediary to help them choose a product. The remaining $1-\mu$ consumers are instead informed and choose the product that maximizes their utility. This setup suits cases of biased prominence, in which uninformed consumers perceive an extra-utility a before purchase, which, however, does not exist after purchase. Accordingly, an important policy question is whether to intervene to mitigate the informational gap.

A possible measure in this direction consists in constraining the intermediary to give equal prominence to the two firms, namely, to send uninformed consumers in $x \leq \frac{1}{2}$ to firm 0 and those in $x > \frac{1}{2}$ to firm 1. This policy is referred to as neutrality (see de Cornière and Taylor, 2019). Hereafter, we focus on the parametric interval in which the market of both types of consumers is fully covered at equilibrium, that is, $t \leq 2\frac{2-\mu}{4-\mu}v$, and investigate how neutrality impacts consumer surplus.

Result 3. *Let $t \leq 2\frac{2-\mu}{4-\mu}v$. After the implementation of a neutrality policy,*

(i) if $t < 2\frac{1-\mu}{3-\mu}v$, the equilibrium configuration remains HD (i.e., x_I and x_I^a still get positive utility); the consumer surplus decreases;

(ii) if $2\frac{1-\mu}{3-\mu}v \leq t < \frac{2}{3}v$, the equilibrium configuration changes from HD to MD on standard (informed) consumers (i.e., x_I now gets zero utility, while x_I^a still gets positive utility); the consumer surplus decreases when $t < \hat{t}$ and increases when $t > \hat{t}$, with $\hat{t} \in (2\frac{1-\mu}{3-\mu}v; \frac{2}{3}v)$.

(iii) if $\frac{2}{3}v \leq t \leq 2\frac{2-\mu}{4-\mu}v$, the equilibrium configuration remains MD on standard/informed consumers (i.e., x_I still gets zero utility and x_I^a still gets positive utility); the consumer surplus increases.

Proof. See Appendix C.2. □

Result 3 shows that the neutrality policy yields two different but intertwined effects. First, the sign of the policy on consumer surplus crucially depends on whether firms interact in HD or in MD. Second, in the interval $2\frac{1-\mu}{3-\mu}v \leq t < \frac{2}{3}v$ the policy triggers a change in the strategic equilibrium behavior of firms, from HD to MD.[11]

These outcomes highlight the importance of accounting for MD when an Hotelling setup is used to draw policy conclusions. If we disregarded the existence of MD, we would be induced to consider that, absent local monopolies, only point (i) of Result 3 applies: the policy compels firms to compete only over the informed consumers; this softens competition and ultimately entails that neutrality is detrimental to consumer surplus. This conclusion is, however, incomplete because point (iii) shows that the policy is actually beneficial to consumers, when the pre-policy equilibrium configuration is MD. At the same time, point (ii) reveals that the policy itself may be the driver of a change in the firms' competition mode, from HD to MD, because the competition-softening effect under HD makes the indifferent standard consumer's utility equal to zero for a lower level of t relative to v. When there is a shift to MD, the take-home of Result 3 is that the

[11]Appendix C.2 elaborates on these effects.

impact on consumers may be reversed because the quasi-collusive behavior of firms under MD neutralizes the competition-softening effect.

Ban on exclusivity. Suppose now that the extra-utility a derives from firm 0's exclusive access either to a superior input that reduces the cost of achieving quality vis-à-vis firm 1 (e.g., Bedre-Defolie and Biglaiser, 2022) or to a premium product (e.g., Martimort and Pouyet, 2022). Unlike the biased intermediary case, here type-a consumers enjoy a after purchasing from firm 0. As a consequence, a possible policy intervention consists in banning exclusive contracts, so rival firms can access the superior input/product as well (see, e.g., Bedre-Defolie and Biglaiser, 2022). In our model, a ban on exclusivity amounts to firm 1 being able to offer the extra-utility a too.

The ensuing result studies how an exclusivity ban impacts consumer surplus under full market coverage. For simplicity, but without loss of generality, we let $\mu = 1$ so that all consumers are type-a, i.e., they are sensitive to (actual) quality. Again, we focus on the parametric interval in which the market is fully covered, i.e., the indifferent consumer obtains nonnegative utility at equilibrium: the interval is $t \leq v + \frac{1}{2}a$, derived by plugging $\mu = 1$ into the upper bound of interval (iv), Result 2.

Result 4. *Let* $t \leq v + \frac{1}{2}a$ *and* $\tilde{t} \equiv \frac{1}{3}(v+a) + \frac{\sqrt{2(2v^2-a^2+4av)}}{6}$. *After the implementation of a ban on exclusivity,*

(i) if $t < \frac{2}{3}v + \frac{1}{3}a$*, the equilibrium configuration remains HD and the consumer surplus increases;*

(ii) if $\frac{2}{3}v + \frac{1}{3}a \leq t < \frac{2}{3}v + \frac{2}{3}a$*, the equilibrium configuration changes from MD to HD; consumer surplus increases if* $t < \tilde{t}$ *and decreases if* $t > \tilde{t}$*.*

(iii) if $\frac{2}{3}v + \frac{2}{3}a \leq t \leq v + \frac{1}{2}a$*, the equilibrium configuration remains MD and the consumer surplus increases.*

Proof. See Appendix C.3. □

In our simple setup, banning exclusivity makes consumers better-off both under HD and MD. However, the implementation of an exclusivity ban can still cause the equilibrium configuration to switch. In this case, point (ii) shows that the shift is from MD to HD and provides a subtle, cautionary warning. In the interval $\tilde{t} < t < \frac{2}{3}v + \frac{2}{3}a$, the consumer surplus jumps *downwards across* configurations, thus questioning the desirability of the policy.

5.3 Quality Competition in Price-Regulated Markets

In several industries, firms do not compete in prices. This occurs because, for instance, prices are regulated by the Government. Firms must therefore revert to other instruments, such as product quality and/or variety, to gain a competitive edge over their rivals. Intuitively, this type of strategic interaction has been largely explored in the spatial competition literature.

Early contributions include Calem and Rizzo (1995), who rely on a Hotelling setup where firms sequentially choose location and qualities, and Gravelle (1999) and Nuscheler (2003), who instead build on the Salop's circular city. Such analyses model the competition between hospitals to attract patients or between public schools/universities to attract students. Brekke et al. (2006) extend Calem and Rizzo (1995) and investigate optimal price regulation. More recently, the Hotelling game with exogenous location and quality competition has been used to to delve into the effects of hospitals' competition on the equilibrium quality levels (e.g., Beitia, 2003, and Esteves et al., 2022) and on the equilibrium quality differences (e.g., Siciliani and Straume, 2019).

In what follows, we modify the baseline model in Section 2 to accommodate quality competition between price-regulated firms. We prove that MD emerges as an equilibrium outcome, like in the standard price-competition Hotelling model, and study how firms' market power affects the quality level. We then exploit the existence of a continuum of asymmetric equilibria under MD to discuss the link between market power and quality differences between firms and to show that price regulation can affect the sign of this link.

The utility of consumer $x \in [0,1]$ if buying from firm i is

$$\mathcal{U}(x, q_i) = 1 + q_i - td_i(x) - p, \tag{11}$$

where p is the price chosen exogenously by a regulator and $1+q_i$ is the gross utility the consumer enjoys from a product/service of quality q_i (for a similar utility function, see Wolinsky, 1997, or Brekke et al., 2006).[12]

Firms simultaneously select the quality level q_i to maximize profit

$$\pi_i(q_i, q_j) = D_i(p - q_i) - \frac{q_i^2}{2}, \tag{12}$$

where D_i is firm i's demand. Offering a service quality level q_i comes at fixed convex cost $\frac{q_i^2}{2}$ and also involves a marginal cost q_i (for a similar profit function, see Brekke et al., 2006, or Siciliani and Straume, 2019).

We solve the quality competition game in the following

Result 5. *Consider quality competition. Three alternative equilibrium configurations arise depending on the level of t relative to p:*

(i) if $t < \sqrt{\left(p - \frac{1}{4}\right)^2 + 1} - \left(p - \frac{1}{4}\right)$, Hotelling Duopoly with symmetric equilibrium qualities $q^{HD} \equiv \max\{0, \frac{p-t}{2t+1}\}$;

(ii) if $\sqrt{\left(p - \frac{1}{4}\right)^2 + 1} - \left(p - \frac{1}{4}\right) \leq t \leq \sqrt{p^2+2} - p$, Monopolistic Duopoly with equilibrium qualities $q_i^{MD} \equiv \min\{p - 1 + \frac{t}{2} + k, t\}$, $q_j^{MD} \equiv \max\{0, p - 1 + \frac{t}{2} - k\}$, and k low enough;

(iii) if $t > \sqrt{p^2+2} - p$, Local Monopolies with symmetric equilibrium qualities $q^{LM} \equiv \max\{0, \frac{2p-1}{t+2}\}$.

Proof. See Appendix D.1. □

The foregoing result shows that MD is not an outcome restricted to price competition and supports the idea that MD is an intrinsic feature of the Hotelling setup. Furthermore, the multiplicity of MD equilibria extends to quality levels as well. Note that the degree of asymmetry

[12] Because firms are price-regulated, here we omit the dependence of $\mathcal{U}(\cdot)$ on p.

in the asymmetric equilibria is upperly bounded by $\min\{\bar{k}, \hat{k}\}$, where $\bar{k} \equiv \frac{4(p-1)t+2t^2-2}{2(2t+3)}$ and $\hat{k} \equiv \frac{2-t^2-2pt}{2(t+2)}$. Finally, q^{HD} is negatively affected by t, while q^{MD} is positively so. Invoking Remark 3, this demonstrates a negative relationship between market power and the equilibrium quality levels supplied by the firms.

Competition, regulation, and quality dispersion. Because asymmetric quality equilibria arise under MD, we are able to investigate the relationship between market power and quality differences, without assuming ex-ante asymmetries between firms.

We consider the range of k that can sustain a MD equilibrium, i.e., $[0, \min \bar{k}, \hat{k}]$, as a metric of *quality dispersion*. If this range expands (shrinks), then the expected quality difference, $q_i^{MD} - q_j^{MD} = 2k$, increases (reduces). We state the following

Result 6. *Consider the MD equilibria in the quality competition game and let* $\tilde{p} \equiv \frac{5+4t-3t^2-2t^3}{7t+4t^2}$ *. If the regulated price p is relatively low, $p < \tilde{p}$, an increase in firms' market power decreases quality dispersion,* $\partial \left(\min\left\{\bar{k}, \hat{k}\right\}\right)/\partial t > 0$. *If the regulated price p is relatively high, $p > \tilde{p}$, an increase in firms' market power increases quality dispersion,* $\partial \left(\min\left\{\bar{k}, \hat{k}\right\}\right)/\partial t < 0$.

Proof. See Appendix D.2. □

The main message conveyed by Result 6 is the following: at the MD equilibrium, a price regulator, by suitably setting the price level, can affect the sign of the relationship between market power and the expected difference in the quality levels supplied by the firms.

6 Conclusion

This paper has thoroughly analyzed the three equilibrium configurations that can arise in the simplest Hotelling model when, for given $v > 0$, t can take any positive value. While the Hotelling Duopoly and Local Monopolies configurations are well-known, Monopolistic Duopoly has attracted far less attention in the literature, perhaps because of its peculiar characteristics: firms interact strategically, but do not actually compete.

We have argued that the existence of MD is due to the sharp increase in the price-cost margin that would be lost by firms if they tried to expand their sales by stealing consumers from each other.

We have also suggested that MD can be interpreted as a form of quasi-collusion. This has potential antitrust implications. Consider a market for horizontally differentiated products, in which similar competitors are active, but prices are observed to be different and/or to move in opposite directions. If the attention is restricted to HD, one might conclude that firms are colluding. Our analysis shows that no collusion is taking place, instead, if firms are operating under MD. As a result, the identification of the actual equilibrium configuration, either HD or MD, is crucial to drive antitrust responses.

We have finally considered three recent and relevant topics in the Hotelling literature and shown how (the inclusion of) MD can dramatically affect both positive and normative results.

A final question concerns the robustness of our analysis. The applications in Section 5 demonstrate that the basic message of our analysis holds in setups that are more sophisticated than the baseline model presented in Section 2. Beyond these extensions, it is easy to prove the following. (i) The results of Proposition 1 continue to hold under more general cost functions such as $C(q) = cq + F$, as long as c and F are not exceedingly large.[13] (ii) The presence of three firms, located in 0, $\frac{1}{2}$ and 1, as in, e.g. Bacchiega and Garella (2022), does not alter the message of the present analysis either. In this case, the boundary between HD and MD is at $t = \frac{4}{3}v$, that separating MD from LM is at $t = 2v$, and equilibrium multiplicity with asymmetric prices may arise under MD. (iii) Assuming a quadratic transportation cost function $t(d_i(x))^2$ does not qualitatively affect our results: if $t < \frac{4}{5}v$, the unique equilibrium features HD; if $\frac{4}{5}v < t < \frac{4}{3}v$ at least one symmetric MD equilibrium exists (with possibly a continuum of asymmetric ones); if $t > \frac{4}{3}v$ the only equilibrium is LM. (iv) Finally, by virtue of the main Proposition in Cremer and Thisse (1991), the robustness of MD to quadratic costs entails that MD also emerges under vertical differentiation, provided that a suitable specification of the Mussa and Rosen (1978) setup is adopted.

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[13] This property follows directly from Salop (1979) analysis, where the boundaries between HD, MD and LM also depend on the marginal production cost c; interestingly, the MD prices do not, which is referred to as "short run price rigidity" by Salop (1979, p. 155).

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Appendices

A Best-response Strategy and Equilibrium

A.1 Proof of Lemma 1

Optimal price response under HD. First, suppose firm 0 chooses p_0 such that $x_I < 1$ at the solution to (4). This yields $p_0 = \frac{p_1+t}{2}$. Substituting this value into (2) and then solving $x_I < 1$ for p_1 gives $p_1 < 3t$. Note that $p_0 = \frac{p_1+t}{2}$ is an admissible solution, provided that the constraint in (4) is fulfilled. Plugging $p_0 = \frac{p_1+t}{2}$ into (3) and then solving $x_0 > x_1$ for p_1 yields $p_1 < \frac{4}{3}v - t$. Summing up, $p_0 = \frac{p_1+t}{2}$ is firm 0 best response under HD if $p_1 < \min\left\{3t, \frac{4}{3}v - t\right\}$.

Then, consider the case $t < \frac{1}{3}v$. If $3t \leq p_1 \leq v$ and firm 0 reacted by charging $p_0 = \frac{p_1+t}{2}$, the last consumer buying from firm 0 would be located at 1 and would get strictly larger utility than when purchasing from firm 1. Firm 0 prefers to set a higher p_0 such that the two utilities are equal and the last consumer located at 1 is also the indifferent one: p_0 solves equation $\mathcal{U}(1, p_0) = \mathcal{U}(1, p_1)$ and is given by $p_1 - t$.

Optimal price response under MD. Price response (7) simply stems from the solution of equation $x_0 = x_1$ for p_0. Note that $x_1 \leq 0$ if $p_1 \leq v - t$, in which case price response (7) makes firm 0 serve a zero demand. Instead, for any $p_1 > v - t$, the demand accruing to firm 0 is given by $\frac{p_1-(v-t)}{t}$.

Optimal price response under LM. Assuming that firm 0 chooses p_0 such that $x_0 < 1$ at the solution to (8), one gets the optimal monopoly price $p_0 = \frac{v}{2}$. This is an admissible solution provided that the constraint in (8) is fulfilled. Substituting $p_0 = \frac{v}{2}$ into (3) and then solving $x_0 < x_1$ for p_1 yields $p_1 > \frac{3}{2}v - t$. Note that the demand accruing to firm 0, $x_0 = \frac{v}{2t}$, is lower than 1 if $t > \frac{1}{2}v$.

Best-response strategy. We consider the three relevant intervals involving t relative to v.

(i) If $t < \frac{1}{3}v$, only the HD and the MD price responses are feasible in $p_1 \in (0, v]$. We compute firm 0's profits as a function of p_1 under HD,

$$\pi_0^{HD}(p_1) = \begin{cases} \frac{(t+p_1)^2}{8t} & \text{if } 0 < p_1 < 3t, \\ p_1 - t & \text{if } 3t \leq p_1 \leq v. \end{cases}$$

and under MD,

$$\pi_0^{MD}(p_1) = \begin{cases} 0 & \text{if } 0 < p_1 \leq v - t, \\ \frac{(2v-t-p_1)(p_1-v+t)}{t} & \text{if } v - t < p_1 \leq v. \end{cases}$$

One can ascertain that $\pi_0^{HD}(p_1) > \pi_0^{MD}(p_1)$ for any $p_1 \in (0, v]$. As a consequence, (5) is the best-response strategy in $p_1 \in (0, v]$.

(ii) If $\frac{1}{3}v \leq t \leq \frac{1}{2}v$, HD and MD price responses are feasible in $p_1 \in \left(0, \frac{4}{3}v - t\right)$, and, again, one can verify that $\pi_0^{HD}(p_1) > \pi_0^{MD}(p_1)$. By contrast, MD is the only feasible price response in $p_1 \in \left[\frac{4}{3}v - t, v\right]$. As a consequence, (5) and (7) are the best-response strategies in $p_1 \in \left(0, \frac{4}{3}v - t\right)$ and $p_1 \in \left[\frac{4}{3}v - t, v\right]$, respectively.

(iii) If $t > \frac{1}{2}v$, HD and MD price responses are feasible in $p_1 \in \left(0, \frac{4}{3}v - t\right)$, and $\pi_0^{HD}(p_1) > \pi_0^{MD}(p_1)$. MD is the only feasible price response in $p_1 \in \left[\frac{4}{3}v - t, \frac{3}{2}v - t\right]$. Finally, MD and LM price responses are feasible in $p_1 \in \left[\frac{3}{2}v - t, v\right]$ and it is straightforward to check that $\pi_0^{LM} = \frac{v^2}{4t} > \pi_0^{MD}(p_1)$. As a consequence, (5), (7), and (9) are the best-response strategies in $p_1 \in \left(0, \frac{4}{3}v - t\right)$, $p_1 \in \left[\frac{4}{3}v - t, \frac{3}{2}v - t\right]$, and $p_1 \in \left(\frac{3}{2}v - t, v\right]$, respectively.

Summing up, the best reply is $p_0^{HD}(p_1)$ for any $t > 0$ and any price $p_1 < \min\{v, \frac{4}{3}v - t\}$. If $t > \frac{1}{3}v$, then $\min\{v, \frac{4}{3}v - t\} = \frac{4}{3}v - t$: for any price $p_1 > \frac{4}{3}v - t$, $p_0^{HD}(p_1)$ cannot be the best reply any more and is replaced by $p_0^{MD}(p_1)$. If $t > \frac{1}{2}v$, then $\min\{v, \frac{3}{2}v - t\} = \frac{3}{2}v - t$: for any price $p_1 > \frac{3}{2}v - t$, $p_0^{MD}(p_1)$ cannot be the best reply any more and is replaced by $p_0^{LM}(p_1)$.

A.2 Proof of Proposition 1

Firm symmetry implies that both firms act as either Hotelling duopolists, monopolistic duopolists, or local monopolies at the Nash equilibrium of the price competition game. In what follows, we verify (whether and) under which parametric conditions these configurations arise at equilibrium.

(i) Assume both firms act as Hotelling duopolists. Solving the system of best responses, $p_0^{HD}(p_1)$ as in (5) and the symmetric one $p_1^{HD}(p_0)$, yields the pair of prices $p_0^{HD} = p_0^{HD} = t$. This is an equilibrium if t is lower than both $3t$, which is trivially true, and $\frac{4v}{3} - t$, which is true for $t < \frac{2}{3}v$.

(ii) Assume both firms act as monopolistic duopolists. Solving the system of best responses, $p_0^{MD}(p_1)$ as in (7) and the symmetric one $p_1^{MD}(p_0)$, yields a continuum of prices p_0^{MD} and p_1^{MD} such that $p_0^{MD} + p_1^{MD} = 2v - t$. The symmetric pair of prices, $p_0^{MD} = p_1^{MD} = v - \frac{t}{2}$, is an equilibrium if $v - \frac{t}{2} \in \left[\frac{4}{3}v - t, \frac{3}{2}v - t\right]$, which gives $t \in \left[\frac{2}{3}v, v\right]$. Any asymmetric pair of prices such that $p_i^{MD} = v - \frac{t}{2} - k$, $p_j^{MD} = v - \frac{t}{2} + k$, and $k > 0$ is an equilibrium if $p_i^{MD} \geq \frac{4}{3}v - t$, which is guaranteed by $k \leq \frac{t}{2} - \frac{v}{3}$, and if $p_j^{MD} \leq \frac{3}{2}v - t$, which is guaranteed by $k \leq \frac{v-t}{2}$.

(iii) Assume both firms act as local monopolists, meaning that $p_0^{LM} = p_1^{LM} = \frac{v}{2}$. This is an equilibrium if $\frac{v}{2}$ is higher than $\frac{3}{2}v - t$, which is true for $t > v$, and lower than v, which is trivially true.

Alternative proof. We provide an alternative proof of Proposition 1. The spirit of this proof will be then used to prove Results 1 to 6 in the ensuing appendices.

(i) First consider HD and anticipate that the indifferent consumer will enjoy a positive utility at the Nash equilibrium:

$$\mathcal{U}(x_I\left(\mathbf{p}^{HD}\right), p_i^{HD}) > 0, \tag{A1}$$

where $\mathbf{p}^{HD} \equiv (p_0^{HD}, p_1^{HD})$ denotes the equilibrium prices under condition (A1). Since such condition implies the market is fully covered, the profit function of firm i for a price vector $\mathbf{p}$, $\pi_i = p_i\left(\frac{1}{2} - \frac{p_i - p_j}{2t}\right)$ is maximized at $p^{HD} \equiv t$. Substituting this value back into condition (A1) yields $\mathcal{U}(x_I\left(\mathbf{p}^{HD}\right), p^{HD}) = v - \frac{t}{2} - t > 0 \Leftrightarrow t < \frac{2}{3}v$.

(ii) Move now to LM. Here, the indifferent consumer obtains a negative utility at equilibrium:

$$\mathcal{U}(x_I\left(\mathbf{p}^{LM}\right), p_i^{LM}) < 0, \tag{A2}$$

with $\mathbf{p}^{LM}$ denoting the equilibrium prices under (A2). Plugging the LM optimal prices $p^{LM} \equiv \frac{v}{2}$ into (A2) yields $\mathcal{U}(x_I\left(\mathbf{p}^{LM}\right), p^{LM}) = v - \frac{t}{2} - \frac{v}{2} < 0 \Leftrightarrow t > v$.

(iii) Last, focus on MD and check whether profitable deviations from the MD equilibrium prices p_i^{MD} and p_j^{MD} exist.

We first consider the higher-price firm j. A unilateral deviation violates the zero-utility condition of the indifferent consumer and must therefore lead to a configuration other than MD. A downward deviation to a price lesser than p_j^M leads to HD. In this case, firm j profit is maximized at $p_j^{D,HD} = \frac{t+2v-2k}{4}$. Note that $p_j^{D,HD} \geq p_j^{MD}$ iff $k \leq \frac{t}{2} - \frac{v}{3}$, in which case there is no profitable downward deviation. An upward price deviation by firm j is easily dealt with since it must lead to LM, where the optimal price is $p^{LM} = \frac{v}{2}$. Note, however, that $p^{LM} < p_j^{MD}$ iff $k > \frac{t-v}{2}$, which is true for $t \in \left[\frac{2}{3}v, v\right]$: an upward deviation by the lower-price firm j is not feasible in the interval of interest.

We now turn to the lower-price firm i and restrict our attention to the parameter constellation $k \leq \frac{t}{2} - \frac{v}{3}$ where firm j has no profitable deviation. Following a downward price deviation that leads to HD, firm i profit is maximized at $p_i^{D,HD} = \frac{t+2v+2k}{4}$. Note, however, that $p_i^{D,HD} > p_i^{MD}$ in the interval under scrutiny, which implies that a downward deviation by the lower-price firm i is not feasible. Finally, an upward price deviation leads to LM, under which the optimal price is $p^{LM} = \frac{v}{2}$. Note that $p^{LM} \leq p_i^{MD} \Leftrightarrow k \leq \frac{v-t}{2}$, in which case there is no profitable upward deviation.

B Two-sided Markets

We proceed in two steps. First we prove points (ii) to (iv) of Result 1, then we prove point (i).

B.1 Proof of Result 1, (ii)-(iv)

HD. Solving for x equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$ as in (10) yields the location of the indifferent consumer,

$$\hat{x} = \frac{1}{2} + \frac{p_1 - p_0 + \alpha(\mathbb{E}(n_0) - \mathbb{E}(n_1))}{2t}. \tag{B1}$$

As the producers have correct expectations about the other side's participation, we can write $\mathbb{E}(D_0) = \hat{x}$ and $\mathbb{E}(D_1) = (1 - \hat{x})$. As the consumers have correct expectations too, we can write $\mathbb{E}(n_i) = \gamma\mathbb{E}(D_i)$. Solving the system of producers' and consumers' expectations, we get $\mathbb{E}(n_0) = \gamma\hat{x}$ and $\mathbb{E}(n_1) = \gamma(1 - \hat{x})$. Plugging these values into $\hat{x}$ and solving for x yields

$$x_I = \frac{1}{2} - \frac{p_0 - p_1}{2(t - e)}. \tag{B2}$$

Platform 0 and platform 1 simultaneously solve problems $\max_{p_0} p_0 x_I$ and $\max_{p_1} p_1(1-x_I)$, respectively. The solution to this game is $p_0^{HD} = p_1^{HD} = p^{HD} \equiv t-e$, with $x_I(p^{HD}) = \frac{1}{2}$ and $n_0(p^{HD}) = n_1(p^{HD}) = \frac{\gamma}{2}$. It follows that the platform equilibrium profits are $\frac{t-e}{2}$ and positive iff $t > e$. Finally, for the indifferent consumer to obtain a positive utility, we need $\mathcal{U}(x_I(p^{HD}), p^{HD}, \mathbb{E}(n_0(p^{HD})) = v + \frac{e}{2} - (t-e) - \frac{t}{2} > 0 \Leftrightarrow t < e + \frac{2}{3}v$.

LM. Exploiting symmetry, we analyze platform 0 only. We solve equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = 0$ for x to get the location of the consumer indifferent between purchasing from platform 0 and not purchasing, $x_0 = \frac{v+\alpha\mathbb{E}(n_0)-p_0}{t}$. Given that the expectations must be consistent with equilibrium participation, we obtain $D_0 = \frac{v-p_0}{t-e}$. Accordingly, platform 0 solves: $\max_{p_0} p_0\left(\frac{v-p_0}{t-e}\right)$. We get $p^{LM} \equiv \frac{v}{2}$ with $x_0(p^{LM}) = \frac{v}{2(t-e)}$ and $x_0(p^{LM}) = \frac{v}{2(t-e)}$. Finally, for the indifferent consumer to obtain a negative utility, we need $\mathcal{U}(x_I(p^{LM}), \mathbb{E}(n_0)(p^H)) = v + e\frac{v}{2(t-e)} - \frac{v}{2} - \frac{t}{2} < 0 \Leftrightarrow t > e + v$.

MD. The indifferent consumer obtains zero utility if $v + (e-t)\left(\frac{1}{2} - \frac{p_0-p_1}{2(t-e)}\right) - p_0 = 0$. Solving for $p_0 + p_1$ yields

$$p_0 + p_1 = 2v - t + e. \tag{B3}$$

At the candidate symmetric equilibrium, we get $p^{MD} \equiv v - \frac{t}{2} + \frac{e}{2}$. To check this is an equilibrium, we exploit symmetry and investigate deviations by platform 0, given that platform 1 sets $p_1 = p^{MD}$.

Deviation to HD. Plugging $p_1 = p^{MD} = \frac{2v-t+e}{2}$ into (B1) yields

$$\frac{2v+t+e}{4t} - \frac{p_0}{2t} + \frac{\alpha\left(\mathbb{E}(n_0) - \mathbb{E}(n_0)\right)}{2t}. \tag{B4}$$

Assuming the deviation by platform 0 leads to HD, the optimal deviation profit is maximized at $p_0^{D,HD} = \frac{2v+t+e+2\alpha(\mathbb{E}(n_0)-\mathbb{E}(n_1))}{4}$. Plugging $p_0^{D,HD}$ into (B4), we find the indifferent consumer's location on the deviation path, $x_I^{D,HD} = \frac{2v+t+e+2\alpha(\mathbb{E}(n_0)-\mathbb{E}(n_1))}{8t}$, and, in turn, the deviation profits, $\pi_0^{D,HD} = \frac{(2v+t+e+2\alpha(\mathbb{E}(n_0)-\mathbb{E}(n_1)))^2}{32t}$. One can check that $\pi_0^{D,HD} > \pi^{MD}$, so the deviation is profitable. However, it is not feasible if the utility of the indifferent consumer is nonpositive, that is, if

$$2\alpha\mathbb{E}(n_0) + 6\alpha\mathbb{E}(n_1) \leq -2v + 3t + 3e. \tag{B5}$$

To further investigate the above condition, we proceed as follows. Among all the possible expectations that can be considered off the equilibrium path, we take rational expectations (i.e., such that $\mathbb{E}(n_0) = \gamma x_I^{D,HD}$ and $\mathbb{E}(n_0) = \gamma(1 - x_I^{D,HD}))$ and plug them into (B5), thus obtaining $-2\alpha\left[\mathbb{E}(n_0) - \mathbb{E}(n_1)\right] \leq 2v + t + e + \frac{2t}{e}(-2v + 3t - 3e)$: the LHS is negative, whereas the RHS is nonnegative iff

$$\frac{(2t-e)(3t-2v-e)}{e} \geq 0, \tag{B6}$$

which is fulfilled given that $t \geq e + \frac{2}{3}v$. This proves there exist expectations off the equilibrium path such that the deviation to HD is not feasible.

Deviation to LM. The deviation price is $p^{LM} \equiv \frac{v}{2}$. Plugging it into (B1), we find the location of the indifferent consumer on the deviation path, $\frac{t+v+e+2\alpha\mathbb{E}(n_0)-2\alpha\mathbb{E}(n_1)}{4t}$. The deviation is not feasible if the utility of the indifferent consumer is nonnegative, that is, iff

$$\alpha\left(\mathbb{E}(n_0)+\mathbb{E}(n_1)\right) \geq \frac{t+e-v}{2}. \tag{B7}$$

Using rational expectations (i.e., such that $x_0 = 1 - x_1 = \frac{v}{2(t-e)}$), the above inequality can be rewritten as $(t+e)(v-t+e) \geq 0$, which is true because $t \leq e+v$. This proves there exist expectations off the equilibrium path such that the deviation to LM is not feasible.

B.2 Proof of Result 1, (i)

We observe that if $t \leq e$, the demand of platform i as in (B2) would increase with p_i, which is implausible. To compute the indifferent consumer's location, we proceed as follows. At a MD symmetric equilibrium, producers' rational expectations should be such that each platform attracts half of the consumers: $\mathbb{E}(D_i) = \frac{1}{2}$ that, in turn, implies $n_i = \frac{\gamma}{2}$. Therefore, rational expectations by consumers imply $\mathbb{E}(n_i) = \frac{\gamma}{2}$. Plugging $\frac{\gamma}{2}$ into (10) and solving for x equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$, we obtain $x_I = \frac{1}{2} + \frac{p_1 - p_0}{2t}$. The indifferent consumer obtains zero utility it $v + \frac{e}{2} - p_0 - t\left(\frac{1}{2} + \frac{p_1-p_0}{2t}\right) = 0$. Solving the above equation for $p_0 + p_1$ yields (B3). This implies that the candidate symmetric equilibrium price is as above, $p^{MD} \equiv \frac{2v-t+e}{2}$, and that we can rely on conditions (B6) and (B7) to check whether it is an equilibrium when $t \leq e$. (B6) turns out to be fulfilled iff (i) $t \leq \frac{e}{3}$; (ii) $\frac{e}{3} < t \leq \frac{e}{2}$ and $v \geq \frac{3t-e}{2}$; (iii) $\frac{e}{2} < t \leq e$ and $v \leq \frac{3t-e}{2}$. As for (B7), note the LHS equals e if $\mathbb{E}(n_0) + \mathbb{E}(n_1) = \gamma$, in which case it is strictly higher than the RHS. As a result, there exist (sufficiently high) expectations off the equilibrium path such that condition (B7) holds.

C Asymmetric Competition

C.1 Proof of Result 2

First, we need to define two threshold locations. While standard consumers purchasing decisions depend on the cutoffs (2) and (3), the type-a indifferent consumer locates at

$$x_I^a(p_0, p_1) = \frac{a + t - p_0 + p_1}{2t},$$

while the marginal type-a consumer for firm 0 is $x_0^a = \frac{v+a-p_0}{t}$. Anticipating that $p_0 \geq p_1$ at any equilibrium, we assume $a \leq t$, which is sufficient to have $x_I^a \leq 1$ at any equilibrium.

Note that for any price vector (p_0, p_1), the utility of the standard indifferent consumer,

$$v - \frac{t + p_0 + p_1}{2}, \tag{C1}$$

is always lower than the utility of the type-a indifferent consumer, $v + \frac{a}{2} - \frac{t+p_0+p_1}{2}$. Consequently, we have five possible equilibrium scenarios. In the following, we check the conditions under which these scenarios arise.

(i) Consumers at x_I and x_I^a obtain positive utility. In this case, firms' profits are $\pi_0 = (1-\mu)\, p_0 x_I + \mu p_0 x_I^a$ and $\pi_1 = (1-\mu)\, p_1 (1 - x_I) + \mu p_1 (1 - x_I^a)$, with equilibrium prices $p_0 =$

$t+\frac{1}{3}a\mu$ and $p_1=t-\frac{1}{3}a\mu$. Plugging these values into (C1) returns the equilibrium utility of the indifferent standard consumer, which is positive iff $t<\frac{2}{3}v$. In such interval, the equilibrium utility of the consumer x_I^a is positive *a fortiori*. Therefore, this is an equilibrium iff $t<\frac{2}{3}v$.

(ii) Consumer at x_I obtains zero utility, while x_I^a obtains positive utility. In this case, the symmetric equilibrium price is $v-\frac{t}{2}$. The equilibrium location of the indifferent standard consumer is $\frac{1}{2}$ and her utility is zero. As for the type-a consumer, we have $\frac{1}{2}+\frac{a}{2t}$ and $\frac{a}{2}$, respectively. Similarly to all the symmetric MD equilibria derived in the present appendix, one can show that no profitable unilateral deviations exist in the interval $\frac{2}{3}v\leq t\leq 2\frac{2-\mu}{4-\mu}v$. The proof follows the same steps as the one in the baseline model, and is left to the reader.

(iii) Consumer at x_I obtains negative utility, while x_I^a obtains positive utility. In this case, firms' profits are $\pi_0=(1-\mu)\,p_0x_0+\mu p_0x_I^a$ and $\pi_1=(1-\mu)\,p_1x_1+\mu p_1(1-x_I^a)$, where x_0 and x_1 are the marginal consumers in (3) and equilibrium prices are p_0^{LM} and p_1^{LM}, as in Result 2. The equilibrium location of the indifferent standard consumer is $x=\frac{4t-t\mu-2a\mu}{8t-2t\mu}$ and her utility is negative iff $t>2\frac{2-\mu}{4-\mu}v$. As for the type-a consumer, we have $x=\frac{4a-4t+4v-3a\mu+t\mu-2v\mu}{8-6\mu}$ and positive utility if $t<2\frac{2-\mu}{4-\mu}v+\frac{4-3\mu}{4-\mu}a$. Concluding, this equilibrium exists iff $t\in\left(2\frac{2-\mu}{4-\mu}v,2\frac{2-\mu}{4-\mu}v+\frac{4-3\mu}{4-\mu}a\right)$.

(iv) Consumer at x_I obtains negative utility, while x_I^a obtains zero utility. The symmetric equilibrium price is $p=v-\frac{t-a}{2}$. The equilibrium location of the indifferent type-a consumer is $\frac{1}{2}+\frac{a}{2t}$ and her utility is zero. This equilibrium configuration is robust to deviations iff $t\in\left[2\frac{2-\mu}{4-\mu}v+\frac{4-3\mu}{4-\mu}a,v+\left(1-\frac{\mu}{2}\right)a\right]$.

(v) Consumers at x_I and x_I^a obtain negative utility. Firm profits are $\pi_0=(1-\mu)\,p_0x_0+\mu p_0x_0^a$ and $\pi_1=(1-\mu)\,p_1(1-x_1)+\mu p_1(1-x_1)$. Equilibrium prices are $p_0=\frac{v+a\mu}{2}$ and $p_1=\frac{v}{2}$. To confirm the equilibrium existence, we only need to check that the utility of the indifferent type-a consumer, located at $x_I^a=\frac{1}{2}+\frac{a(2-\mu)}{4t}$, is negative: this is true iff $t>v+\left(1-\frac{\mu}{2}\right)a$.

In conclusion, we are interested in computing the equilibrium consumer surplus in the interval $t\leq 2\frac{2-\mu}{4-\mu}v$. In doing so, we remove a from the utility function of type-a consumers because a is not enjoyed *ex post*. We get $CS_{HD}\equiv v-\frac{5}{4}t-\frac{a^2\mu(9-4\mu)}{36t}$ and $CS_{MD}\equiv\frac{t}{4}-\frac{a^2\mu}{4t}$.

C.2 Proof of Result 3

The neutrality policy limits competition to the market of standard consumers. The indifferent type-a consumer is *exogenously* located at $\frac{1}{2}$.

Equilibrium analysis after policy.

(i) Assume that the indifferent standard and type-a consumers obtain positive equilibrium utility. Profits are then $\pi_0=(1-\mu)\,p_0x_I+\mu p_0\frac{1}{2}$ and $\pi_1=(1-\mu)\,p_1\,(1-x_I)+\mu p_1\frac{1}{2}$. The symmetric equilibrium price is $\frac{t}{1-\mu}$. The equilibrium location of both types of indifferent consumers is $\frac{1}{2}$ and their utility is positive iff $t<2\frac{1-\mu}{3-\mu}v$.

(ii) Focus now on $t>2\frac{1-\mu}{3-\mu}v$ and assume that the indifferent standard and type-a consumers obtain negative equilibrium utility. Profits are then $\pi_0=p_0x_0$ and $\pi_1=p_1(1-x_1)$.

The symmetric optimal price is $\frac{v}{2}$. The equilibrium location of both types of indifferent consumers is $\frac{1}{2}$ and their utility is negative iff $t > v$. Recall that we are focusing on $t \leq 2\frac{2-\mu}{4-\mu}v$ and note that $2\frac{2-\mu}{4-\mu}v < v$.

(iii) Finally, consider $2\frac{1-\mu}{3-\mu}v \leq t \leq 2\frac{2-\mu}{4-\mu}v$ and assume that the indifferent standard and type-a consumers obtain zero equilibrium utility. In this case, the symmetric equilibrium price is $v - \frac{t}{2}$, the equilibrium location of both types of indifferent consumers is $\frac{1}{2}$ and their utility is zero.

Consumer surplus after policy. After the policy, the consumer surplus under HD is $CS_{HD,P} \equiv v - \frac{5-\mu}{4(1-\mu)}t$. It is a matter of algebra to ascertain that this value is lower than CS_{HD}. The post-policy consumer surplus under MD is $CS_{MD,P} = \frac{t}{4}$, which is instead higher than CS_{MD}.

Consumer surplus comparison with configuration change. We focus on the interval $2\frac{1-\mu}{3-\mu}v \leq t \leq \frac{2}{3}v$, where firms shift from HD to MD after the policy, and compare the resulting consumer surplus, CS_{HD} and $CS_{MD,P}$. We first observe that CS_{HD} is monotonically decreasing in t and that $CS_{MD,P}$ is monotonically increasing in t. We also have that $CS_{HD} - CS_{MD,P} > 0$ at $t = 2\frac{1-\mu}{3-\mu}v$ and $CS_{HD} - CS_{MD,P} < 0$ at $t = \frac{2}{3}v$. Let then $\hat{t} \in \left(2\frac{1-\mu}{3-\mu}v, \frac{2}{3}v\right)$ denote the unique threshold where $CS_{HD} - CS_{MD,P} = 0$.

Explanation of Result 3. The neutrality policy is CS-reducing under HD and CS-enhancing under MD for the following reasons. The policy eliminates the bias: this enhances the consumer surplus, irrespective of whether the configuration is HD or MD. Under HD, however, the policy softens the incentive to steal consumers from the rival because firms can only compete to attract the informed consumers. The lower resulting competitive pressure on firms reduces the consumer surplus. If HD is the equilibrium configuration before and after the policy implementation, the negative competition-softening effect outweights the positive no-bias effect, making consumers worse-off. If MD is the pre- and post-policy equilibrium configuration, there is no competition-softening effect because firms set the same equilibrium price $p = v - \frac{t}{2}$ before and after the policy implementation. Put it differently, the quasi-collusive behavior adopted by firms under MD excludes the possibility to further soften competition. As a result, the only force at work is the bias-elimination one, which makes consumer surplus to increase.

The change in the firm strategic behavior from HD to MD is due to the competition-softening effect under HD, which reduces consumers' utility. As a result, the indifferent, informed consumer's utility becomes zero, thus giving rise to MD, for a lower level of t relative to v, i.e., $2\frac{1-\mu}{3-\mu}v$ rather than $\frac{2}{3}v$.

C.3 Proof of Result 4

The ban on exclusivity enables firm 1 too to offer the extra utility a.

Equilibrium analysis before policy with $\mu = 1$. We have

(i) HD, i.e., the indifferent (type-a) consumer gets positive utility, with $p_0 = t + \frac{1}{3}a$, $p_1 = t - \frac{1}{3}a$ and $t < \frac{2}{3}v + \frac{1}{3}a$;

(ii) MD, i.e., the indifferent consumer obtains zero utility, with $p = v - \frac{t-a}{2}$ and $\frac{2}{3}v + \frac{1}{3}a \leq t \leq v + \frac{1}{2}a$;

(iii) LM, i.e., the indifferent consumer obtains negative utility, with $p_0 = \frac{v+a}{2}$ and $p_1 = \frac{v}{2}$ and $t > v + \frac{1}{2}a$.

Consumer surplus before policy. We compute the equilibrium consumer surplus in the case of full market coverage, i.e., if $t \leq v + \frac{1}{2}a$. We include a in the computation of surplus because a is actual utility. We get $CS'_{HD} \equiv v - \frac{5}{4}t + \frac{a(a+18t)}{36t}$ and $CS'_{MD} \equiv \frac{a^2+t^2}{4t}$.

Equilibrium analysis after policy. After the ban on exclusivity, firms are identical.

(i) Under HD, the indifferent consumer is as in (2). The equilibrium prices are t, the equilibrium location of the indifferent consumer is $\frac{1}{2}$ and her utility is positive iff $t < \frac{2}{3}(v+a)$.

(ii) Assume that $t > \frac{2}{3}(v+a)$ and that the indifferent consumer obtains negative equilibrium utility, such that the configuration is LM. Profits are $\pi_0 = p_0 \frac{v+a-p_0}{t}$ and $\pi_1 = p_1 \frac{v+a-p_1}{t}$. The symmetric optimal price is $\frac{v+a}{2}$, the equilibrium location of the indifferent consumer is $\frac{1}{2}$ and her utility is negative, if and only if, $t > v + a$. Recall that we are focusing on $t \leq v + \frac{1}{2}a$ and note that $v + \frac{1}{2}a < v + a$.

(iii) Finally, consider $\frac{2}{3}(v+a) \leq t \leq \left(v + \frac{1}{2}a\right)$; this interval is nonempty under assumption $a < t$. Assume that the indifferent consumer obtains zero equilibrium utility, which implies MD. In this case, the symmetric equilibrium price is $v - \frac{t-a}{2}$ and the equilibrium location of the indifferent consumer is $\frac{1}{2}$.

Consumer surplus after policy. Under HD, we have $CS'_{HD,P} \equiv v - \frac{5}{4}t + a$ and, under MD, $CS'_{MD,P} \equiv \frac{1}{2}a + \frac{1}{4}t$. It is a matter of simple algebra to ascertain that $CS'_{HD,P} > CS'_{HD}$ and $CS'_{MD,P} > CS'_{MD}$.

Consumer surplus comparison with configuration change. We focus on the interval $\frac{2}{3}v + \frac{1}{3}a \leq t < \frac{2}{3}(v+a)$, where firms shift from MD to HD after the policy and compare the resulting consumer surplus, CS'_{MD} and $CS'_{HD,P}$. We have $CS'_{MD} \leq CS'_{HD,P}$ if $\frac{2}{3}v + \frac{1}{3}a \leq t \leq \tilde{t}$ and $CS'_{MD} > CS'_{HD,P}$ if $\tilde{t} < t < \frac{2}{3}(v+a)$, where $\tilde{t} \equiv \frac{1}{3}(v+a) + \frac{\sqrt{2(2v^2-a^2+4av)}}{6}$.

D Quality Competition

D.1 Proof of Result 5

The indifferent consumer is $\mathcal{U}(x, q_0) = \mathcal{U}(x, q_1) \Leftrightarrow x = \frac{t+(q_0-q_1)}{2t} \equiv x_I(\mathbf{q})$, where the boldfaced letter $\mathbf{q}$ represents the quality vector (q_0, q_1). The marginal consumers are found by setting (11) equal to zero and solving for x, $\mathcal{U}(x_i, q_i) = 0 \Leftrightarrow x_i(q_i) = \frac{1-p+q_i}{t}$. Hence, the firms' demands are $D_0(\mathbf{q}) = x_I(\mathbf{q})$ and $D_1(\mathbf{q}) = 1 - x_I(\mathbf{q})$ if $x_1(q_1) \leq x_I(\mathbf{q}) \leq x_0(q_0)$, while they are $D_0(q_0) = x_0(q_0)$ and $D_1(q_1) = 1 - x_1(q_1)$ if $x_0(q_0) < x_I(\mathbf{q}) < x_1(q_1)$.

HD. The demands are $D_i(\mathbf{q})$. By taking the first-order derivative of (12) w.r.t. q_i, setting it to zero and solving form q_i we get firms' best response functions. The solution to the system of best replies is $q_i = \frac{p-t}{2t+1}$. By plugging the optimal qualities back into (12), we obtain the profits at the HD equilibrium of the game, $\frac{t(t+1)(4p+1)-p^2}{2(2t+1)^2}$. The equilibrium utility of the indifferent consumer is positive iff

$$t < \sqrt{\left(p-\frac{1}{4}\right)^2+1}-\left(p-\frac{1}{4}\right). \tag{D1}$$

Note that the HD equilibrium quality is negative if $t > p$ and the HD equilibrium profit is negative if $t < \frac{(2p+1)\sqrt{4p+1}}{2(4p+1)} - \frac{1}{2}$. In these cases, we assume that firms sets $q_i = 0$ and the regulator $p = 0$. The indifferent consumer's equilibrium utility is then $1 - \frac{t}{2}$, which is positive under condition (D1) with $p = 0$.

LM. The demands are $D_i(q_i)$. Profit maximization yields $q_i = \frac{2p-1}{t+2}$. At the optimal quality level, the profit is $\frac{ptv+4v^2-p^2t}{t(t+16)}$. The equilibrium utility of the indifferent consumer is negative iff $t > \sqrt{p^2+2} - p$.

Note that the optimal LM quality is negative if $p < \frac{1}{2}$ and the LM optimal profit is negative if $p > 1 \cup t < \frac{1}{2p(p-1)}$. In these cases, we assume that firms sets $q_i = 0$ and the regulator $p \in (1 - \frac{t}{2}, 1)$, with the effect that firms' profit is positive but lower than $\frac{ptv+4v^2-p^2t}{t(t+16)}$, and the indifferent consumer's equilibrium utility is negative.

MD. Setting the utility of the indifferent consumer equal to zero and solving for $q_0 + q_1$, we obtain $q_0 + q_1 = 2(p-1) + t$. Like in point (ii) of Proposition 1, we define the MD qualities as

$$q_i = p-1+\frac{t}{2}+k, \quad q_j = p-1+\frac{t}{2}-k, \tag{D2}$$

with $k \geq 0$. At these qualities, the firms' profits are $\pi_i = \frac{2(t+2k)(2-t-2k)-t(2p-2+t+2k)^2}{8t}$ and $\pi_j = \frac{2(t-2k)(2-t+2k)-t(2p-2+t-2k)^2}{8t}$. We check under which conditions there are no profitable deviations from the MD candidate equilibria (D2).

Let us start by considering the low-quality firm j. An upward quality deviation leads to HD and the deviation profit is maximized for $q_j = \frac{2k+4p-t-2}{4(t+1)}$. This deviation is ruled out if the resulting quality is lower than $p - 1 + \frac{t}{2} - k$, which is the case iff $k < \frac{4(p-1)t+2t^2-2}{2(2t+3)} \equiv \bar{k}$. A downward deviation to LM requires that $\frac{2p-1}{t+2} < p - 1 + \frac{t}{2} - k$, which is never verified in the MD interval.

Move now to the high-quality firm i and consider a downward deviation to LM, where the optimal quality is $\frac{2p-1}{t+2}$. To disregard this deviation, it must be that $\frac{2p-1}{t+2} > p - 1 + \frac{t}{2} + k$, which is true iff $k < \frac{2-t^2-2pt}{2(t+2)} \equiv \hat{k}$. Finally, if firm i deviates upwards to HD, its profit function is maximized for $q_i = \frac{4p-2k-t-2}{4(t+1)}$. Yet, this quality level is lesser than $p - 1 + \frac{t}{2} + k$ in the MD interval $\sqrt{\left(p-\frac{1}{4}\right)^2+1} - \left(p-\frac{1}{4}\right) \leq t \leq \sqrt{p^2+2} - p$; this rules out the deviation.

Note that a sufficient condition for the lower equilibrium quality $q_j = p - 1 + \frac{t}{2} - k$ to be positive is $t > \sqrt{\frac{4-3p}{2}}$ and a sufficient condition for the firm j's equilibrium profit to be positive is $t < \frac{4+8p-9p^2+(3p-2)\sqrt{4-4p+9p^2}}{16p(p-1)}$. If these conditions are violated, we assume that firm j sets $q_j = 0$ and the regulator sets p such that firm i's best response is $q_i = \min\{\bar{k}, \hat{k}\}$. The indifferent consumer's equilibrium location is then either $\min\{\frac{3t^2+2t+2}{6t^2+8t}, 1\}$, if $\bar{k} > \hat{k}$, or $\frac{2t^2+9t-2}{4t^2+12t} < 1$, if $\bar{k} < \hat{k}$.

D.2 Proof of Result 6

Notice that $\bar{k} \gtreqless \hat{k} \Leftrightarrow p \gtreqless \frac{5+4t-3t^2-2t^3}{7t+4t^2} \equiv \tilde{p}$. Moreover $\bar{k} = 0$ for $t = \sqrt{\left(\frac{1}{4} - p\right)^2 + 1} - \left(p - \frac{1}{4}\right)$ and $\frac{\partial \bar{k}}{\partial t} = \frac{1}{2} \frac{12p+12t+4t^2+1}{(2t+3)^2} > 0$, while $\hat{k} = 0$ for $t = \sqrt{p^2 + 2} - p$ and $\frac{\partial \hat{k}}{\partial t} = -\frac{1}{2} \frac{4p+4t+t^2+2}{(t+2)^2} < 0$. Rearranging the MD interval wih respect to p yields $\frac{2+t-2t^2}{4t} \leq p \leq \frac{2-t^2}{2t}$. Simple calculations show that $\tilde{p} \in \left(\frac{2+t-2t^2}{4t}, \frac{2-t^2}{2t}\right)$.